RESEARCH STATEMENT

ÁKOS NAGY

My research interests lie in the intersection of geometric analysis and mathematical physics. The methods I use are analytic, typically coming from elliptic and parabolic PDEs. The problems I study are geometric and are motivated by models in the natural sciences. In particular, I have worked on PDEs that occur in the theories of superconductivity, particle physics, and biophysics. My current research has the following four major directions:

1. Ginzburg–Landau equations (details in Section 1): The Ginzburg–Landau theory is a phenomenological model for superconductivity dating back to the 1950's. It is a variational theory for an abelian gauge field and a complex scalar field, and it is one of the earliest gauge theoretic examples of spontaneous symmetry breaking. The Ginzburg–Landau equations form a system of nonlinear PDEs, and there is a vast literature on them. In the situations I like to consider, the underlying space is a compact Riemannian manifold, typically a surface, which is the case where the physical interpretation is the best-understood. The Ginzburg–Landau equations depend on two coupling parameters, and the existence, behavior of solutions, and moduli spaces change as the parameters vary. In [27, 32], I proved results related to the (non)existence of solutions, that is, the question of when spontaneous symmetry breaking occurs; in [26, 32] I studied moduli problems, that is, the properties of the space of all solutions; and in [32], I constructed novel solutions.

These projects have resulted in two published papers: [26] in the *Communications in Mathematical Physics* and [27] in the *Journal of Geometric Analysis*. Furthermore, I have one submitted paper [32] and one paper in preparation [5].

2. Yang–Mills Instantons and Monopoles (details in Section 2): Yang–Mills theory form the mathematical foundations of the Standard Model of physics and have also been extremely influential in differential geometry and topology since the 1980's. The corresponding Yang–Mills equation, and the related monopole equation, are nonlinear systems of gauge theoretic PDEs.

I study moduli problems in Yang–Mills theory and other gauge theories. The moduli space of a gauge theoretic equation is the set of all of its solutions, up to a natural equivalence relation, called gauge equivalence. I have constructed and described new examples of such moduli spaces, in situations that have theoretical physical relevance.

I also study the asymptotics of the Yang–Mills equations, which involves understanding the behavior of solutions along the ends of noncompact spaces, for example providing sharp decay estimates on large spheres in $\mathbb{R}^n$. This latter problem often aids the former: solutions can often be labeled by their asymptotic behavior, thus yielding a better understanding of the corresponding moduli spaces.

These projects have resulted in five published or accepted papers: [13] in the *Journal of Geometry and Physics*, [28] in *Communications in Analysis and Geometry*, [29] in *Selecta Mathematica*, [30] in the *Letters in Mathematical Physics*, and [4] is accepted in the *Journal of Mathematical Physics*. The last one is a paper with undergraduate students. Furthermore, I have a submitted paper [14] and a paper in preparation [3].

3. Keller–Segel equations (details in Section 3): The Keller–Segel equations provide a model for chemotaxis, that is, the movement of organisms (typically bacteria) in the presence of a (chemical) substance. The simplest Keller–Segel system is a pair of evolution equations on the density of the organisms and the concentration of the substance. The most interesting case (both physically and mathematically) is the 2-dimensional one. While the literature of the theory is vast when the underlying space is the flat plane, $\mathbb{R}^2$, not much is known when the background is curved or topologically nontrivial, despite their obvious importance in applications. In my work [25, 33] I have studied the Keller–Segel system on curved planes and closed surfaces.

This project has resulted in one submitted paper [33], and there is one paper in preparation [25], which is a paper with an undergraduate student.

4. Conjugate linear perturbations of Dirac operators (details in Section 4): In the 1980's, Jackiw and Rossi gave a model for electronic excitations of 2-dimensional superconductors. With the advent of the study of topological phases of matter, this theory drew renewed interest from physicists. Nonetheless the rigorous mathematical formulation of the Jackiw–Rossi equations had not been provided. Recently, I gave an interpretation of the Jackiw–Rossi theory in terms of spin geometry, and generalized it to curved background and higher dimensions, both of which had been missing even from the physics literature. These "generalized" Jackiw–Rossi equations are Dirac type equations, where the Dirac operator is perturbed by a conjugate linear term. I gave a complete classification of such perturbations and I constructed solutions over 2-dimensional complete surfaces.

This project has resulted in one submitted paper [31].

In the next four sections I give a more detailed exposition of these directions, presenting my past work and my future plans. In Section 5, I give an outline on my mentoring work with students and other synergistic activities.

1. Ginzburg–Landau theory; [26,27,31,32]

Let (X, g) be a Riemannian manifold, and $(\mathscr{L}, h)$ be a Hermitian line bundle over X. The *Ginzburg–Landau energy* of a connection, ∇ (with curvature tensor F_∇), and a section, ϕ, of $\mathscr{L}$ is

$$\mathscr{E}_{\mathrm{GL}}(\nabla, \phi) = \int\limits_X \left(|F_\nabla|^2 + |\nabla\phi|^2 - \alpha|\phi|^2 + \tfrac{\beta}{2}|\phi|^4 \right) \mathrm{vol}_g. \tag{1.1}$$

Here α and β are positive coupling constants. The Ginzburg–Landau equations are the Euler–Lagrange equations of the functional (1.1).

My current and past collaborators on these projects are Gonçalo Oliveira (IST Austria) and Da Rong Cheng (UWaterloo).

1.1. **Past work.** In [26], I studied the moduli space, $\mathscr{M}$, of absolute minimizers of (1.1) over closed surfaces. These minimizers are called *vortices*. I proved, using Green's function techniques, that when $\beta = \frac{1}{2}$ and $\alpha \to \infty$, then the tangent vectors of $\mathscr{M}$ have an asymptotic form. Using this I studied the canonical L^2-metric of the vortex moduli space and computed the holonomy of corresponding Berry connection.

In [27], I studied irreducible (that is, when ϕ is nonzero) solutions to the Ginzburg–Landau equations on compact surfaces, potentially with nonempty boundaries. I first proved a gauged version of the Palais–Smale Compactness condition for the functional (1.1). Using this result I then gave necessary conditions to the existence of irreducible solutions, which are also sufficient when $\beta \geqslant \frac{1}{2}$. These irreducible solutions are, in fact, absolute minimizers of the Ginzburg–Landau energy (1.1). The physical interpretation of this result is that these conditions characterize the cases when spontaneous symmetry breaking occurs, that is, when the absolute minimizers have $\phi \neq 0$, thus such solutions are not symmetric with respect to gauge transformations. Finally, I proved, once again using Palais–Smale Compactness, that for any give values of the coupling constants, α and β, the moduli space of critical points of (1.1) is compact.

In [32], with my collaborator, Gonçalo Oliveira, we constructed novel solutions to the Ginzburg–Landau equations using two vastly different methods. The first is topological and uses proof by contradiction: If there were no nontrivial solutions, then the space of all configurations would contract to the space of trivial solutions. By computing the homotopy type of these spaces, we showed that this is impossible. The second method uses an adapted version of the Ljapunov–Schmidt reduction to show that there are special "bifurcation points", such that if the parameter α is close to one of these points, then reducible (ϕ identically zero) solutions bifurcate into irreducible solutions. This latter method works in higher dimensions also. Finally, let me note that solutions are necessarily unstable and are the first examples of such solutions on nontrivial line bundles.

1.2. **Future directions.** In [38], Pigati and Stern showed that, in a certain sense, limits of solutions to the Ginzburg–Landau equations, with large coupling and bounded energy, "cut out" minimal codimension two varifolds. Together with Da Rong Cheng, I am currently working on the converse of this result: we aim to prove that given a codimension two submanifold, $S^{n-2} \subset X^n$, there exists a sequence of solutions to the Ginzburg–Landau equations, concentrating around S, in the sense of Pigati–Stern.

Our method is a glue-in method, using an infinite dimensional version of Lyapunov–Schmidt reduction, that is, even the "reduced" equation is not a finite dimensional one. The idea is to construct a family of approximate solutions via gluing the exact 1-vortex solution on the complex plane to the normal bundle of a submanifold S' that is close to S. We show that for any large enough coupling this gluing is possible and as the coupling goes to infinity S' "converges" to S.

2. Instantons and Monopoles

Given an oriented Riemannian manifold (X, g) and a principal G-bundle P over X, connection, ∇, on P, with curvature, F_∇, is a Yang–Mills connection if it has finite L^2 norm, that is, $|F_\nabla| \in L^2(X, g)$, and satisfies

$$d_\nabla^* F_\nabla = 0. \tag{2.1}$$

This equation is the Euler–Lagrange equation of the Yang–Mills energy which is the square of the L^2-norm of F_∇. The study of Yang–Mills solutions have a long history and here I will only focus on the parts relevant to my research.

Instantons and monopoles can be viewed as special solutions to equation (2.1). They are special for (at least) three different reasons: They satisfy a simpler, first order, elliptic equation (as opposed to a second order one, such as equation (2.1)). They are absolute minimizers of the Yang–Mills energy. They have well-understood physical interpretations.

For the above reasons, the study of the geometric and analytic properties of instantons and monopoles, and their moduli spaces, is central in modern differential geometry and theoretical (particle) physics.

My current and past collaborators on these projects are Benoit Charbonneau (UWaterloo), Anuk Dayaprema (UWisconsin Madison, graduate student), Gábor Etesi (Budapest University of Technology), Daniel Fadel (Peking University), C.J. Lang (UWaterloo, graduate student), Gonçalo Oliveira (IST Austria), Haoyang Yu (Duke University, undergraduate student).

2.1. **Past work.** In [13], Gábor Etesi and I used a mathematically rigorous definition of the abelian Yang–Mills path integral to answer questions about an old conjecture in particle physics, called S-duality. We evaluated these integrals using ζ-function regularization and heat kernel approximations. We found that while the S-duality conjecture does not hold for the classical Yang–Mills energy, there are canonical ways to extend the theory in a way that the resulting partition function is S-dual. Our results hold for closed 4-manifolds and a large class of complete 4-manifolds, including the multi-Taub–NUT spaces.

In [28], Gonçalo Oliveira and I constructed infinitely many new examples of Yang–Mills instantons on the (noncompact) Euclidean Schwarzschild manifold. The Euclidean Schwarzschild manifold is a model for black holes, and, geometrically, it is complete and Ricci-flat, but it is not hyperkähler, thus many important tools, such as the bow construction of Cherkis, are not available. Before our results, only finitely many examples were known for each energy level, and no examples were known for most energies.

In [29], Gonçalo Oliveira and I studied complex monopoles, called Haydys monopoles. We showed the existence of Haydys monopoles on $\mathbb{R}^3$ and constructed an open subset of the corresponding moduli space.

In [30], Gonçalo Oliveira and I proved a nonexistence result for complex instantons, called Kapustin–Witten fields, on certain noncompact, complete Riemannian manifolds, called gravitational instantons.

In [14], Daniel Fadel, Gonçalo Oliveira, and I studied monopoles on G_2-manifolds. This is work is the first installment of a series of papers aimed to study the Donaldson–Segal program; cf. [12]. In Section 2.2, I describe this project in more detail.

In [4], Benoit Charbonneau, C.J. Lang, Anuk Dayaprema, Haoyang Yu, and I used the technique of the Nahm transform to construct novel monopoles on $\mathbb{R}^3$. This project grew out of a summer research program that I led at Duke University with undergraduate students Anuk Dayaprema and Haoyang Yu.

2.2. **Future directions.** I currently pursue two directions in this topic.

The first is a continuation of the work started in [14]. We study G_2-monopoles on noncompact G_2-manifolds, and their relations to coassociative submanifolds.

For our present purposes, a G_2-manifold is a smooth, oriented Riemannian 7-manifold with a compatible 3-form, φ that is closed, coclosed, and nondegenerate. Such a metric is necessarily Ricci-flat. A beautiful introduction to the basic linear algebra and geometry of G_2-manifolds can be found in the thesis of Karigiannis [22]. An important problem in G_2-geometry is to develop methods to distinguish G_2-manifolds and study their special, *calibrated* submanifolds. In [12], Donaldson and Segal suggested a conjectural connection between certain minimal submanifolds, called coassociative submanifolds, and certain gauge theoretic objects, called G_2-monopoles. In this project we study the analytic and geometric properties of the latter. Briefly, G_2-monopoles are pairs of a connection, ∇, on some principal bundle, and a Higgs field, Φ, which is a section of the adjoint bundle, that together satisfy the G_2-monopole equation:

$$*_\varphi (F_\nabla \wedge *_\varphi \varphi) = \nabla\Phi.$$

The study of G_2-monopoles was initiated by Cherkis in [6] and Oliveira in [36, 37]. Oliveira gave the first evidence supporting the Donaldson–Segal program by finding families of G_2-monopoles parametrized by a positive real number $m > 0$, called the mass, which in the limit $m \to \infty$ concentrate along a compact coassociative submanifold.

In [14], together with Daniel Fadel and Gonçalo Oliveira, I showed that several of the asymptotic features satisfied by these examples are in fact general phenomena which follow from natural assumptions such as finiteness of a relevant energy functional, called intermediate energy. This is a very much needed development in order to justify the choice of function spaces to be used in a satisfactory moduli theory. The authors prove their results using ϵ-regularity type results and energy bounds, similar to the ones used in [7].

An obvious next step is investigating the moduli problem. The standard route to prove that the moduli space is a manifold is: 1. Proving that the linearization is Fredholm; 2. Showing that the cokernel is trivial (or, at least, constant rank); 3. Computing the index. With Daniel Fadel and Gonçalo Oliveira, I am preparing a paper showing these statements.

THe second direction concerns BPS monopoles on $\mathbb{R}^3$ and is a joint work with Benoit Charbonneau.

We call a pair (∇, Φ) BPS monopole on $\mathbb{R}^2$, if ∇ is a connection on a principal bundle, P, Φ is a section of the adjoint bundle of P, and together they satisfy

$$F_\nabla = *\nabla\Phi, \quad \& \quad F_\nabla \in L^2.$$

The standard hypothesis in the literature is that (up to gauge) finite energy BPS monopoles converge to smooth pairs $(\nabla_\infty, \Phi_\infty)$ on the *sphere at infinity*, S^2_∞, where ∇_∞ is a Yang–Mills connection and Φ_∞ parallel. If the eigenvalues of Φ_∞ are all distinct, then the monopole (∇, Φ) is said to have *maximal symmetry breaking*. An important tool to study the moduli spaces of monopoles with maximal symmetry breaking is the Nahm transform. In [34], Nahm associated a solution of an ordinary differential equation, *Nahm's equation*, to each monopole. He also gave a method to recover the monopole from this solution. In [17, 35], Hitchin and Nakajima made this method rigorous for $G = SU(2)$. In [18, 19], Hurtubise and Murray showed how to construct monopoles with maximal symmetry breaking for any compact Lie group G.

These pictures provide deep connections between algebraic geometry, analysis, gauge theory, and mathematical physics, but are only well-understood in the case of maximal symmetry breaking. When the rank is greater than 2, irreducible monopoles with nonmaximal symmetry breaking exist, but less is known about these monopoles, much less the corresponding moduli spaces.

Together with Benoit Charbonneau, I developed a generalization of the Nahm transform that applies to monopoles with arbitrary symmetry breaking. We are currently preparing a paper, [3], in which we show that for any type of symmetry breaking there exists Nahm data such that the generated BPS monopole has the given type of symmetry breaking. This paper is the first of a two part series; in the second paper, we show that every BPS monopole, with the above described asymptotics, arises as the Nahm transform of such a Nahm data in a unique (up to gauge) way.

3. Keller–Segel equations

The Keller–Segel type equations describe *chemotaxis*, that is, the movement of organisms (typically bacteria) in the presence of a (chemical) substance. The simplest Keller–Segel system is a pair of equations on the density of the organisms, ϱ, and the concentration of the substance, c, both of which are functions on $[0,\mathrm{T})\times\mathbb{R}^n$. Furthermore, ϱ is assumed to be nonnegative and integrable. Together they satisfy the (parabolic-elliptic) Keller–Segel equations:

$$(\partial_t + \Delta)\varrho = \mathrm{d}^*\big(\varrho\,\mathrm{d}c\big), \tag{3.1a}$$

$$\Delta c = \varrho, \tag{3.1b}$$

where d is the gradient, d^* is its L^2-dual (the divergence), and $\Delta = \mathrm{d}^*\,\mathrm{d}$. The mass of ϱ is

$$m := \int_{\mathbb{R}^d} \varrho(x)\,\mathrm{d}^n x \in \mathbb{R}_+,$$

is a conserved quantity.

The most studied case is the planar one. When the metric is the standard, euclidean metric on $\mathbb{R}^2$, the literature of equations (3.1a) and (3.1b) is vast; for introduction, see [1, 10, 11]. However, little is known about the case when the underlying space is not the (flat) plane. In these projects I study geometrically, or topologically nontrivial backgrounds.

My current and past collaborators on these projects are Israel Michael Sigal (UToronto) and Adam Mendenhall (UC Santa Barbara).

3.1. **Past work.**

In [33], I studied the case when the metric is conformally equivalent to the flat metric and the conformal factor has the form $e^{2\varphi}$, where φ is smooth and compactly supported. Some of the results are novel already in the flat ($\varphi = 0$) case. In particular, I proved that (under very mild hypotheses), that there are no static solutions to equations (3.1a) and (3.1b), unless the mass is 8π. Furthermore, I showed that there are metrics, arbitrarily close to the flat one on the plane, that do not support stationary solutions to the static Keller–Segel equation even when the mass is 8π.

3.2. **Future directions.**

Building on the results of [33], I will further study equations (3.1a) and (3.1b) on curved planes. This work is partially a collaboration with Israel Michael Sigal.

First, I will prove short time existence for the general case. In order to do this, I will adapt the proof from the flat case [1]. The key obstruction of verbatim using the original proof is that one does not explicitly know the Green's function and the heat kernel for an arbitrarily curved plane. Nonetheless, by proving strong enough bounds for these functions, the ideas of [1] can be improved to work in the general case also.

Then I will study the problem of long time existence. On the flat plane, for low mass (less than 8π), and for initial values with finite second moment, solutions to equations (3.1a) and (3.1b) exist for all time. This is proved using an appropriate "virial theorem" that gives control over the second moment for all times. This virial theorem does not generalize in a straightforward manner to curved planes. Using a nontrivial and noncanonical version of the second moment, I have proved a curved virial theorem for masses less than $8\pi - \epsilon$, where ϵ depends on the metric. This yields long time existence for low enough masses. I am currently working on sharpening this result.

I am also working on a project concerning the Keller–Segel equations on closed (compact and without boundary) surfaces. This project is a collaboration with my current senior thesis student, Adam Mendenhall at UC Santa Barbara.

We show that if one considers functions that have analytic Fourier components, that is

$$\varrho(x,t) = \sum_{a} \sum_{n\in\mathbb{N}} \mathrm{R}_{n,\lambda} t^n f_{\lambda_a}(x), \tag{3.2}$$

where λ_a runs through the spectrum of the Laplacian and f_{λ_a} is the corresponding eigenvector, then equations (3.1a) and (3.1b) become an iteration on the components $\mathrm{R}_{n,\lambda}$, that is, $\mathrm{R}_{n,\lambda}$ only depends on coefficients $\mathrm{R}_{m,\mu}$, where $m < n$. The main difficulty in showing that the coefficients exist for all n and they provide a well-defined solution through equation (3.2) is that the iteration involves "triple-products" of eigenfunctions

$$\varphi_{abc} := \int_{\Sigma} f_{\lambda_a} f_{\lambda_b} f_{\lambda_c} \,\mathrm{dA},$$

which are generally hard to compute.

Currently we are working on the cases, when the surface is either a flat torus or a round sphere. In these cases the numbers φ_{abc} are easy to handle (in fact, exactly computable for the torus). Our goal is to show that, under mild hypotheses on the initial data, this iterative method yields solutions to the Keller–Segel equations.

4. Conjugate linear perturbations of Dirac operators

In [21], Jackiw and Rossi introduced a Dirac-type equation on the plane that describes electronic excitation on an s-wave superconductor. An unusual feature of the Jackiw–Rossi equation is that it contains a complex conjugate linear term, hence the solutions only form a real vector space. In fact, ground states of this theory are interpreted as Majorana fermions pinned to vortices [2]. Furthermore, this theory has potential applications in quantum computing; cf. [15, 20, 24].

4.1. **Past work.** In [31], I reformulated the classical Jackiw–Rossi theory in terms of spin geometry and generalized the Jackiw–Rossi (Hamiltonian) operator to more general fields and higher dimensions. Furthermore, I studied the spectral properties of this theory.

These *generalized Jackiw–Rossi operators* have the form $\mathrm{H} = \mathbb{D} + \mathscr{A}$, where $\mathbb{D}$ is a Dirac-type operator on and $\mathscr{A}$ is a conjugate linear bundle map. Since H is not complex linear, its eigenspaces are not complex (but only real) subspaces of the Hilbert space.

I remark, that certain special cases have been studied in the context of pseudo-holomorphic curves; cf. [8, 9, 16, 23, 39, 40]. This connection has the potential to yield further applications of our results, but we do not explore this direction any further in the present work.

My main result in [31] is a construction of the low energy spectrum of H on complete surfaces. The method of the proof was understanding a model case on the flat, complex plane, and then gluing in the model solutions in the general case.

4.2. **Future directions.** As a continuation of this project I will prove similar results to that of [31] in higher dimensions, via first understanding the appropriate model cases there. Beyond the obvious physical motivation, there is another potential use for this: On closed manifolds, H is a compact perturbation of the Dirac operator, thus they have the same Fredholm index. Using this observation and the main theorem of [31], I gave a short and novel proof of the Riemann–Roch theorem. Given a higher dimensional analogue of this result, I plan to produce similarly simple, new proofs of (certain cases of) the Atiyah–Singer theorem.

5. Mentoring and other synergistic activities

I am always looking for students to advise. I have submitted a paper with C.J. Lang (Waterloo), Anuk Dayaprema (UW-Madison), and Haoyang Yu (Duke); see [3]. Furthermore, I am currently preparing a paper for submission with Adam Mendenhall (UCSB).

I have been involved with the organization of the following workshops and meetings:

(1) Geometry and Physics of ALX Metrics in Gauge Theory (workshop), at the American Institute of Mathematics, July 25–29, 2022. Coorganizing with Laura Fredrickson (UOregon), Steve Rayan (USaskatchewan), and Hartmut Weiß (Kiel).
(2) Joint Mathematics Meeting, Seattle, Special Session on "Intersections of geometric analysis and mathematical physics", January 5–8, 2022. Coorganizing with Xianzhe Dai (UC Santa Barbara).
(3) Geometry, Analysis, and Quantum Physics of Monopoles (online workshop), at the Banff International Research Station, January 31–February 5, 2021. Coorganized with Benoit Charbonneau (UWaterloo), Sergey Cherkis (UArizona), and Jacques Hurtubise (McGill).
(4) AMS Fall Eastern Sectional Meeting (online), Special Session on "Recent developments in Gauge Theory", October 3–4, 2020. Coorganized with Siqi He (Simons Center).
(5) AMS Fall Southeastern Sectional Meeting, University of Florida, Special Session on "Geometry of Gauge Theoretic Moduli Spaces", November 2–3, 2019. Coorganized with Chris Kottke (NCF).

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